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ORIGINAL PAPER

A Note on Cyclic Connectivity and Matching Properties of Regular Graphs

Michael D. Plummer · Tao Wang · Qinglin Yu

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Abstract In this paper cyclic connectivity is studied in relation to certain matching properties in regular graphs. Results giving sufficient conditions in terms of cyclic connectivity for regular graphs to be factor-critical, to be 3-factor-critical, to have the restricted matching properties $E(m, n)$ and to have defect-d matchings are obtained.

Keywords Cyclic connectivity · $[r-1, r]$-graph · Factor-critical · Bicritical · Matching extension · Defect-d · Near bipartite

1 Introduction

All graphs considered in this paper will be simple.

A set of edges E in a graph G is called a *cyclic edge-cut* if $G - E$ contains at least two components each of which contains a cycle. The size of any smallest cyclic edge-cut is called the *cyclic edge-connectivity* of G and is denoted by $c\lambda(G)$.

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M. D. Plummer (✉)
Department of Mathematics, Vanderbilt University, Nashville, TN 37240, USA
e-mail: michael.d.plummer@vanderbilt.edu

T. Wang
Institute of Applied Mathematics, College of Mathematics and Information Science,
Henan University, Kaifeng 475004, China
e-mail: wangtao@henu.edu.cn

Q. Yu
Department of Mathematics and Statistics, Thompson Rivers University, Kamloops, BC, Canada
e-mail: yu@tru.ca

Cyclic connectivity has its early roots in the study of graph coloring. For example, Birkhoff [6] proved already in 1913 that in order to prove the Four Color Conjecture, it suffices to prove it for those plane graphs which are cubic and cyclically-5-connected.

Péroche [17] showed that if $|V(G)| \geq 6$, then $c\lambda(G) \leq 3|V(G)| - 3$ and the bound is sharp. In [12] a polynomial algorithm for computing $c\lambda(G)$ for regular graphs is given, although, to the best of our knowledge, the complexity of computing cyclic connectivity for graphs in general remains unknown.

More recently, this graph parameter has also been studied in connection with certain matching properties in graphs. A graph is said to be *k-factor-critical* if the deletion of every set of k vertices results in a graph with a perfect matching. For the values $k = 1$ and $k = 2$, these properties are more commonly called *factor-criticality* and *bicriticality*, respectively. These two classes of graphs are important building blocks in a canonical theory of decomposition for graphs in terms of their maximum matchings. For more on this, as well as a general reference on matching theory in graphs, the reader is referred to [13].

A graph G with at least $2n + 2$ vertices is said to be *n-extendable* if every matching of size n can be extended to (i.e., is a subset of) a perfect matching in G. This concept was formally introduced in [23] and has since given rise to many research papers. Two surveys on n-extendable graphs are [20] and [21].

In [16] (see also [22]) it was shown that r-regular non-bipartite graphs of even order with cyclic connectivity at least $r + 1$ must be bicritical, while r-regular bipartite graphs with cyclic connectivity at least $(n - 1)r + 1$ must be n-extendable. Cyclic connectivity and extendability for planar graphs has also been investigated with the earliest result in this direction [8] stating that every 3-connected 3-regular planar graph with cyclic connectivity at least 5 is 2-extendable.

In the present paper, we generalize some of these results in several different directions. In addition, we will investigate cyclic connectivity in relation to the "restricted" matching properties $E(m, n)$ which have been introduced more recently. Let G be a connected graph with at least $2(m + n + 1)$ vertices which contains a perfect matching. Then G is said to have *property* $E(m, n)$ (or simply, "G is $E(m, n)$") if, for each pair of disjoint matchings $M, N \subseteq E(G)$ with $|M| = m$ and $|N| = n$, there is a perfect matching F in G such that $M \subseteq F$ and $F \cap N = \emptyset$ (thus the property $E(m, 0)$ is exactly the same as the property of being m-extendable). The property $E(m, n)$ was first introduced in [24] and has since been further investigated in [1] and in the series of papers [2]–[5].

We use $o(G)$ to denote the number of odd components of G, and $E(S, T)$, the set of the edges between vertex sets S and T. Denote by $\nabla(S)$ the set of edges with exactly one endvertex in the set S. We call a graph G an $[r-1, r]$-*graph*, if every vertex is of degree r or $r - 1$. Finally, we denote the minimum degree of graph G by $\delta(G)$.

2 Preliminary Results

Lemma 2.1 *Let G be a graph and S, a subset of $V(G)$. If $G[S]$ contains no cycles, then* $|\nabla(S)| \geq (\delta(G) - 2)|S| + 2$.

Proof Since by hypothesis $G[S]$ contains no cycles, it must be a forest and hence $|E(G[S])| \leq |S| - 1$. Consequently, $|\nabla(S)| \geq \delta(G)|S| - 2(|S| - 1) = (\delta(G) - 2)|S| + 2$. □

Corollary 2.2 *Let G be a graph with $\delta(G) = \delta \geq 2$ and S, a non-null proper subset of $V(G)$. Let $\sigma = \min\{|S|, |V(G) - S|\}$. Then $|\nabla(S)| \geq \min\{(\delta - 2)\sigma + 2, c\lambda(G)\}$.*

Proof Let $C = \nabla(S)$. If $G[S]$ and $G[V(G)-S]$ both contain cycles then $|C| \geq c\lambda(G)$ and the assertion holds. Thus we may assume that either $G[S]$ or $G[V(G) - S]$ contains no cycles. Assume without loss of generality that $G[S]$ contains no cycles. Then by Lemma 2.1, $|C| \geq (\delta - 2)|S| + 2 \geq (\delta - 2)\sigma + 2$, where the last inequality follows from the hypothesis that $\delta \geq 2$.

□

Corollary 2.3 *Let G be a graph with $\delta(G) \geq 2$. Then $\lambda(G) = \min\{\delta, c\lambda(G)\}$.*

Proof Let $C = \nabla(S)$ be a non-empty edge-cut in G. By Corollary 2.2, we have that $|C| \geq \min\{\delta, c\lambda(G)\}$. Thus $\lambda(G) \geq \min\{\delta, c\lambda(G)\}$. But since $\lambda(G) \leq \delta$ and $\lambda(G) \leq c\lambda(G)$ by definition, the result follows. □

The following result is clear.

Lemma 2.4 *Suppose $S \subseteq V(G)$, $|S|$ is odd and each vertex in S has degree r. Then $|\nabla(S)| \equiv r \pmod 2$.*

□

3 Cyclic Connectivity, $E(m, n)$ and k-Factor-Criticality

We begin with a result due to Plesník [19].

Theorem 3.1 *If G is an r-regular graph of even order and $\lambda(G) \geq r - 1$, then G contains a perfect matching that excludes any $(r - 1)$ given edges.*

Corollary 3.2 *Suppose G is an r-regular graph of even order and $c\lambda(G) \geq r - 1$. Then G has a perfect matching which excludes any $r - 1$ given edges.*

Proof Assume that $c\lambda(G) \geq r - 1$. If $r = 1$, the assertion trivially holds, so suppose $r \geq 2$. Then by Corollary 2.3, $\lambda(G) \geq r - 1$ and the result then follows by Theorem 3.1. □

Our first result involving $E(m, n)$ is also an easy consequence of Theorem 3.1.

Corollary 3.3 *Suppose G is an r-regular graph of even order and $c\lambda(G) \geq r - 1$. Then G is 1-extendable* [*i.e.*, *G is $E(1, 0)$*].

Remark 3.4 The condition $c\lambda(G) \geq r - 1$ in Corollary 3.3 is sharp. For each $r \geq 2$, Plesník provided graphs which show this. In fact, these graphs do not even contain a perfect matching.

Fig. 1 4-regular, but not 1-extendable

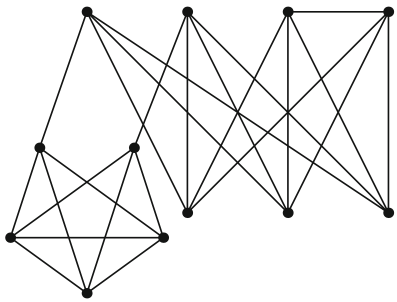
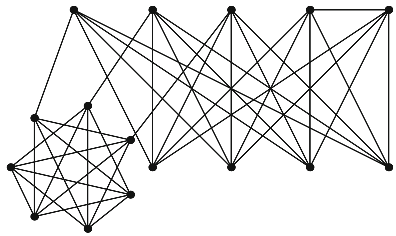


Fig. 2 5-regular, but not 1-extendable



Theorem 3.5 ([18]). *Suppose* $0 \leq k \leq r-2$. *Then there exists an* r*-regular graph* G *of even order such that*

$$\lambda(G) = \begin{cases} k+1, & \text{if } r \text{ is even and } k \text{ is odd;} \\ k, & \text{otherwise} \end{cases}$$

and such that G *does not contain a perfect matching.*

It is also possible to provide another class of graphs which *do* contain a perfect matching, but also show the sharpness of the bound on $c\lambda(G)$ in Corollary 3.3.

For r even, let $G' = (X, Y)$ be the complete bipartite graph $K_{r,r-1}$ with $X = \{x_1, x_2, \ldots, x_r\}$, and K be the graph obtained from K_{r+1} by removing a matching $\{u_1u_2, u_3u_4, \ldots, u_{r-3}u_{r-2}\}$. Then join the edges x_iu_i for $i = 1, 2, \ldots, r-2$, and add edge $x_{r-1}x_r$ to yield a graph G_r. The graph G_r is r-regular and $c\lambda(G_r) = r-2$, but G_r is not 1-extendable. See an example for $r = 4$ (see Fig. 1).

For r odd, Let $G' = (X, Y)$ be the complete bipartite graph $K_{r,r-1}$ with $X = \{x_1, x_2, \ldots, x_r\}$, let K be a graph obtained from K_{r+2} by removing the edges on the path $u_1u_2 \cdots u_r$ and an edge $u_{r+1}u_{r+2}$. Join $u_{i+1}x_i$ for $i = 1, 2, \ldots, r-2$ and add edge $x_{r-1}x_r$. Then we get a graph G'_r. Clearly, G'_r is r-regular, and $c\lambda(G'_r) = r-2$, but G'_r is not 1-extendable. See an example for $r = 5$ (see Fig.2).

Corollary 3.6 *Suppose* G *is an* r*-regular graph of even order and* k *is a non-negative integer with* $k \leq r-1 \leq (|V(G)|-2)/2$. *Then if* $c\lambda(G) \geq r-1$, G *is* $E(0, k)$.

Remark 3.7 The assumption $r-1 \leq (|V(G)|-2)/2$ in Corollary 3.6 is a natural one since property $E(0, r-1)$ is only defined for $r-1 \leq (|V(G)|-2)/2$.

Remark 3.8 The reader is reminded that $E(0, n) \not\Longrightarrow E(0, n-1)$ for $n \geq 4$. However, $E(0, 3) \Longrightarrow E(0, 2) \Longrightarrow E(0, 1) \Longrightarrow E(0, 0)$ (see Theorems 4.4.1 and 4.4.2 in [14]).

Remark 3.9 The assumption that $c\lambda(G) \geq r-1$ in Corollary 3.6 above is best possible by Corollary 2.3 and Theorem 3.1 above.

In part (i) of the next theorem, since G is assumed to have odd order, then $r \geq 2$ and the result follows by Corollary 2.3 and Theorem 1 of [19]. Part (ii) is just Theorem 3.3 of [22] (note that the requirement that $r \geq 3$ is necessary for an r-regular graph to be bicritical).

Theorem 3.10 *Let G be an r-regular graph. Then the following two properties hold.*

(i) *If G has odd order and $c\lambda(G) \geq r-1$, then G is factor-critical.*
(ii) *If $r \geq 3$, G has even order, is non-bipartite and $c\lambda(G) \geq r+1$, then G is bicritical.*

Now let us consider the property $E(1, 1)$. It is known (cf. Theorem 4.2 in [23]) that a 2-extendable graph is either bipartite or bicritical (clearly no graph can be both) and also 2-extendability implies the property $E(1, 1)$. On the other hand, it is well-known that the properties of being bicritical and being $E(1, 1)$ are independent in that neither implies the other. However, each of these two properties implies 1-extendability.

Theorem 3.11 *If $r \geq 3$ and G is a connected r-regular graph of even order with $c\lambda(G) \geq r+1$, then either* (a) *G is $E(1, 1)$ or* (b) *G contains two edges e and f such that every perfect matching of G which contains e also contains f and $G-e-f$ is a 1-extendable bipartite graph.*

Proof Suppose first that G is bipartite. Then G is $E(1, 1)$ by Theorem 3.4 of [3]. So suppose that G is non-bipartite (then, by Theorem 3.10, G is bicritical and hence 1-extendable). Assume further that G is not $E(1, 1)$. Thus there are two independent edges e and f in $E(G)$ such that every perfect matching in G containing e also contains f.

Then by Tutte's theorem there exists a vertex set $S \subseteq V(G)$ containing the endvertices of e, such that $o(G-f-S) \geq |S|$. But then, since G is 1-extendable, it follows that $o(G-f-S) = |S|$ and the edge f connects two odd components of $G-f-S$.

Assume now that the odd components of $G-f-S$ are $O_1, O_2, \ldots, O_s$, where $|\nabla(O_1)| \leq |\nabla(O_2)| \leq \cdots \leq |\nabla(O_s)|$ and $s = |S|$. Since $\lambda(G) = r$ by Corollary 2.3, we have that $|\nabla(O_1)| \geq r$. But then since G is r-regular, it follows that $|\nabla(O_i)| = r$, for $1 \leq i \leq s$. Let $s_i = \min\{|V(O_i)|, |V(G) - V(O_i)|\}$, for $i = 1, \ldots, s$. By Corollary 2.2, we have then that $r = |\nabla(O_i)| \geq \min\{(r-2)s_i + 2, c\lambda(G)\}$ and hence $s_i = 1$ for $1 \leq i \leq s$.

Thus every O_i is a singleton and it follows that $G-f-S$ has no even components, $G[S]-e$ is independent and $G-e-f$ is a connected bipartite graph with bipartition $(S, V(G)-S)$.

Note then that the edge set $E(G) - \{e, f\}$ is an edge-cut in G (separating e from f).

It remains to show that $G-e-f$ is 1-extendable. Suppose, to the contrary, that $G-e-f$ is not 1-extendable. So there must exist an edge g in $G-e-f$ such that no perfect matching in $G - e - f$ contains g. Then arguing as above, with the edge pair $\{e, g\}$ replacing the pair $\{e, f\}$, we have, in particular, that $E(G) - \{e, g\}$ is also an edge-cut in G. But it is well-known (cf. [26]) that the symmetric difference of edge-cuts is again an edge-cut and in this case this symmetric difference is precisely $\{f, g\}$. So $\lambda(G) \leq 2$. But G is 3-edge-connected by Corollary 2.3, since $r \geq 3$. This contradiction completes the proof. □

Remark 3.12 A 1-extendable non-bipartite graph G is said to be *near bipartite* if it contains edges e and f such that $G-e-f$ is bipartite and 1-extendable. These graphs have arisen in the study of Pfaffian graphs (Cf. [7], [9], [15]).

We next proceed toward a result linking cyclic connectivity and 3-factor-criticality.

Theorem 3.13 *Suppose $r \geq 4$. Let G be an r-regular graph of odd order. If $c\lambda(G) > 2r$ and G is not 3-factor-critical, then there is a set $S \subseteq V(G)$ such that $|V(G)-S| = |S| - 1$ and $V(G) - S$ is independent.*

Proof Suppose that $r \geq 4$, $c\lambda(G) > 2r$, but G is not 3-factor-critical. Then by Tutte's theorem, G contains a set S with $|S| \geq 3$ such that $o(G - S) \geq |S| - 2$. But by hypothesis, G has odd order and hence

$$o(G - S) \geq |S| - 1. \tag{1}$$

Let S be a maximal set satisfying inequality (1). Then $G - S$ has no even components and each odd component is factor-critical. Let O be any odd component of $G - S$ that is not a singleton.

We claim that $|E(O, S)| \geq 2r + 2$. By Theorem 5.5.1 of [13] component O must contain a cycle. If $G - V(O)$ also contains a cycle, then $|E(O, S)| \geq c\lambda(G) \geq 2r+1$. But then by Lemma 2.4, $|E(O, S)| \geq 2r+2$. On the other hand, if $G-V(O)$ does not contain a cycle, since S contains at least three vertices and G has odd order, it follows that $|V(G) - V(O)|$ is even. But then by Lemma 2.1, $|E(O, S)| \geq 4(r - 2) + 2 = 4r - 6 \geq 2r + 2$, where the last inequality follows from the fact that $r \geq 4$. But in both cases we have $|E(O, S)| \geq 2r + 2$ as claimed.

Now let t denote the number of odd components of $G - S$ that are not singletons. Then $r \cdot o(G - S) + (r + 2) \cdot t \leq |\nabla(S)| \leq r|S|$. Thus

$$o(G - S) \leq |S| - t - 2t/r. \tag{2}$$

From inequalities (1) and (2) we deduce that $t = 0$. Therefore, $V(G) - S$ is independent. But G has odd order, so from (1) and (2) we have that $o(G - S) = |S| - 1$. But then $V(G - S)$ is independent and consists of precisely $|S| - 1$ vertices.

At this point we describe some families of bipartite graphs which will be denoted by $\Sigma_{k,m,n}$. We say that a bipartite graph has an (a, b)-bipartition if it has a bipartition (X, Y) such that $|X| = a$ and $|Y| = b$. We define the following classes $\Sigma_{k,m,n}$ of bipartite graphs for all positive integers k, m and n with $2k \leq m \leq n$. If $k = 1$, then $\Sigma_{1,m,n}$ is the collection of all trees with an (m, n)-bipartition. If $k \geq 2$, then a bipartite graph G belongs to $\Sigma_{k,m,n}$ if and only if there exists an (m, n)-bipartition (X, Y) of

G with $|X| = m$ and $|Y| = n$ and a subset Z of X with $|Z| = 2k - 1$ such that the subgraph of G induced by $Y \cup Z$ is a complete bipartite graph (i.e., isomorphic to $K_{2k-1,n}$) and each vertex in $X \setminus Z$ has degree one in G. It is easy to see that each graph in $\Sigma_{k,m,n}$ has $(2k - 1)(n - 1) + m$ edges, but does not have k independent cycles.

We shall need the following result due to Wang.

Theorem 3.14 [25]. *Let $G = (V_1, V_2; E)$ be a bipartite graph with $2k \le m = |V_1| \le |V_2| = n$, where k is a positive integer. Suppose that the number of edges of G is at least $(2k - 1)(n - 1) + m$ and G does not belong to $\Sigma_{k,m,n}$. Then G contains k independent cycles.*

We shall also need the next result due to Lou and Holton.

Theorem 3.15 ([11]). *Suppose G is a connected r-regular graph with girth g, then $c\lambda(G) \le g(r - 2)$.*

Theorem 3.16 *Suppose $r \ge 4$ and that G is an r-regular graph of odd order. Suppose further that*

$$c\lambda(G) > \max\{2r, 3(r - 2), 2(r - 2)|V(G)|/r\}.$$

Then G is 3-factor-critical.

Proof Note that since G is r-regular of odd order, r must be even. Suppose G is not 3-factor-critical. Thus by Theorem 3.13, there is a set $S \subseteq V(G)$ such that $|V(G)-S| = |S| - 1$ and $V(G) - S$ is a set of independent vertices. Moreover, each vertex of $V(G) - S$ is adjacent to r members of S. Now form a bipartite subgraph H of G by deleting all edges having both endvertices in S. This bipartite subgraph has bipartition $(S, V(G)-S)$ where $|V(G)-S| = |S|-1$. Let $|S| = n$ and $|V(G)-S| = m = n-1$. Finally, let $r = 2k$.

Case 1 Suppose $m \ge r = 2k$.

Then there are rm edges between S and $V(G)-S$. But then $rm = (2k-1)(n-1)+m$ and H does not belong to $\Sigma_{k,m,n}$ for $k \ge 2$. So by Theorem 3.14, there are $k = r/2$ vertex-disjoint cycles in H. Hence

$$c\lambda(G) \le 2(r - 2)|V(G)|/r,$$

a contradiction.

Case 2 So suppose $m < r = 2k$.

Clearly, $|S| = r = 2k$. Therefore, since H is simple, it is a complete bipartite graph $K_{r,r-1}$. Since $m = o(G - S) \ge 2$, it follows that $|S| = m + 1 = 2k \ge 4$, and hence there are at least two independent edges in $G[S]$; i.e., there are at least two vertex-disjoint triangles in G. But then by Theorem 3.15, $c\lambda(G) \le 3(r - 2)$, a contradiction and the proof is complete.

Note that the preceding result fails to hold when $r = 2$, for just let G be a pentagon.

4 Cyclic Connectivity and Defect-*d* Matchings

We now turn our attention to defect-d matchings.

A matching is called a *defect-d matching* if it covers exactly $|V(G)| - d$ vertices of G. Clearly, a defect-0 matching is a perfect matching.

Theorem 4.1 ([10]). *Let G be a graph and n, d non-negative integers such that $n + d + 2 \leq |V(G)|$ and $|V(G)| - n - d \equiv 0 \pmod 2$. Then, for any n-vertex set $T \subseteq V(G)$, $G - T$ has a defect-d matching if and only if*

$$o(G - S) \leq |S| - n + d \quad \text{for any } S \subseteq V(G) \quad \text{with } |S| \geq n.$$

We now use Theorem 4.1 to obtain the following result.

Theorem 4.2 *Let G be a connected graph with $|V(G)| - n - d \equiv 0 \pmod 2$. If $r \geq 3$ and G is an $[r-1, r]$-graph with exactly k vertices of degree $r - 1$, $c\lambda(G) \geq r - 1$ and $r(d + 2 - n) \geq k + 1$, then for any n-vertex set T, $G - T$ has a defect-d matching.*

Proof Suppose not. By Theorem 4.1, there exists a set $S \subseteq V(G)$ with $|S| \geq n$ such that $o(G - S) > |S| - n + d$, and hence $o(G - S) \geq |S| - n + d + 2$ by parity. Let $|S| = m$ and the odd components of $G - S$ be $O_1, O_2, \ldots, O_p$, where $p \geq m - n + d + 2$. Without loss of generality, assume $|E(O_1, S)| \leq |E(O_2, S)| \leq \cdots \leq |E(O_p, S)|$.

If $|E(O_1, S)| \leq r - 2$, then $|O_1| \geq 3$, since the minimum degree of G is greater than $r - 2$. Suppose that O_1 is a tree. Then O_1 must send at least $2(r - 2)$ edges to S and hence $2r - 4 \leq r - 2$ and $r \leq 2$, a contradiction.

Therefore, O_1 is not a tree and hence it contains a cycle. Similarly, the subgraph $G - O_1$ also contains a cycle and so $c\lambda(G) \leq r - 2$. But this contradicts the assumption that $c\lambda(G) \geq r - 1$.

So we assume that $|E(O_1, S)| \geq r - 1$. Suppose the first q odd components of $G - S$ each have exactly $r - 1$ edges incident with S, and the others have at least r edges incident with S. Then $O_1, O_2, \ldots, O_q$ each contain a vertex of degree $r - 1$, for otherwise, $\sum_{v \in O_i} d_{O_i}(v) = r|O_i| - (r - 1) = r(|O_i| - 1) + 1$ is odd, a contradiction. Thus $q \leq k$. It is easy to see that there are at least $(r - 1)q + r(p - q)$ edges joining the odd components to S. However,

$$\begin{aligned}(r - 1)q + r(p - q) &= rp - q \geq r(m - n + d + 2) - k \\ &= rm + r(d + 2 - n) - k \geq rm + 1,\end{aligned}$$

which contradicts the fact that $|\nabla(S)| \leq mr$. This completes the proof. □

Remark 4.3 By setting $n = 1$ and $d = k = 0$ in the above result, we have an independent proof of Theorem 3.10(i).

By setting $n = 2$, we also have the following result:

Corollary 4.4 *Suppose $r \geq 3$. Let G be a connected $[r - 1, r]$-regular graph with exactly k vertices of degree $r - 1$ and suppose $|V(G)| - 2 - d \equiv 0 \pmod 2$. If $c\lambda(G) \geq r - 1$ and $rd \geq k + 1$, then for any two distinct vertices $u, v \in V(G)$, $G - \{u, v\}$ has a defect-d matching.*

5 Concluding Remarks

In this paper, we have investigated the restricted matching properties $E(m, n)$, the k-factor-criticality property and the existence of defect-d matchings in graphs under certain cyclic connectivity conditions. By Lou and Holton's construction in [11], large cyclic connectivity does not guarantee 2-extendability. So the only possible values for $E(m, n)$ available for study in this regard are $E(1, k)$ and $E(0, k)$, for $k \geq 0$.

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